1. Primes less than 200

2. Perfect squares less than 1,000
   \[
   \begin{array}{cccccccc}
   0^2 = 0 & 5^2 = 25 & 10^2 = 100 & 15^2 = 225 & 20^2 = 400 & 25^2 = 625 & 30^2 = 900 \\
   1^2 = 1 & 6^2 = 36 & 11^2 = 121 & 16^2 = 256 & 21^2 = 441 & 26^2 = 676 & 31^2 = 961 \\
   2^2 = 4 & 7^2 = 49 & 12^2 = 144 & 17^2 = 289 & 22^2 = 484 & 27^2 = 729 \\
   3^2 = 9 & 8^2 = 64 & 13^2 = 169 & 18^2 = 324 & 23^2 = 529 & 28^2 = 784 \\
   4^2 = 16 & 9^2 = 81 & 14^2 = 196 & 19^2 = 361 & 24^2 = 576 & 29^2 = 841 
   \end{array}
   \]

3. Powers of natural numbers
   \[
   \begin{array}{cccccccc}
   2^0 = 1 & 3^0 = 1 & 4^0 = 1 & 5^0 = 1 & 6^0 = 1 & 7^0 = 1 & 8^0 = 1 & 9^0 = 1 \\
   2^1 = 2 & 3^1 = 3 & 4^1 = 4 & 5^1 = 5 & 6^1 = 6 & 7^1 = 7 & 8^1 = 8 & 9^1 = 9 \\
   2^2 = 4 & 3^2 = 9 & 4^2 = 16 & 5^2 = 25 & 6^2 = 36 & 7^2 = 49 & 8^2 = 64 & 9^2 = 81 \\
   2^3 = 8 & 3^3 = 27 & 4^3 = 64 & 5^3 = 125 & 6^3 = 216 & 7^3 = 343 & 8^3 = 512 & 9^3 = 729 \\
   2^4 = 16 & 3^4 = 81 & 4^4 = 256 & 5^4 = 625 & 6^4 = 1296 & 7^4 = 2401 \\
   2^5 = 32 & 3^5 = 243 & 4^5 = 1024 \\
   2^6 = 64 & 3^6 = 729 \\
   2^7 = 128 \\
   2^8 = 256 \\
   2^9 = 512 \\
   2^{10} = 1024 
   \end{array}
   \]

4. Fractions and Decimal Equivalents
   \[
   \begin{array}{cccc}
   \frac{1}{2} = .5 & \frac{1}{2} = .15 & \frac{1}{2} = .09 \\
   \frac{1}{3} = \frac{3}{5} & \frac{1}{3} = .83 & \frac{1}{3} = .18 \\
   \frac{2}{3} = \frac{3}{5} & \frac{2}{3} = .125 \\
   \frac{4}{5} = .75 & \frac{4}{5} = .375 \\
   \frac{1}{5} = .2 & \frac{1}{5} = .625 \\
   \frac{2}{5} = .4 & \frac{2}{5} = .875 \\
   \frac{3}{5} = .6 & \frac{3}{5} = .1 \\
   \frac{4}{5} = .8 & \frac{4}{5} = .2 
   \end{array}
   \]

5. Factorials
   \[
   \begin{array}{cccc}
   0! = 1 \\
   1! = 1 \\
   2! = 2 \\
   3! = 6 \\
   4! = 24 \\
   5! = 120 \\
   6! = 720 \\
   7! = 5040 \\
   8! = 40,320 \\
   9! = 362,880 \\
   10! = 3,628,800 
   \end{array}
   \]

Mathcounts
Things to Know
6. **Combinatorics Formulas**
   \[ nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!} \text{ (when order does not matter)} \]
   \[ nP_r = \frac{n!}{(n-r)!} \text{ (when order does matter)} \]

7. **Divisibility rules**
   2 → if units digit is even
   3 → if sum of digits is divisible by 3
   4 → if last two digits form a number divisible by 4
   5 → if units digit is 0 or 5
   6 → if number is divisible by both 2 and 3
   7 → if result of subtracting twice the last digit from the number remaining when the last digit is removed is divisible by 7
   8 → if last three digits of number form a number divisible by 8
   9 → if sum of digits is divisible by 9
   10 → if units digit is 0
   11 → if result of alternately adding and subtracting the digits is divisible by 11.
   12 → if number is divisible by both 4 and 3.

8. **Special Factorizations**
   \[ a^2 + 2ab + b^2 = (a + b)^2 \]
   \[ a^2 - 2ab + b^2 = (a - b)^2 \]
   \[ a^3 + 3a^2b + 3ab^2 + b^3 = (a + b)^3 \]
   \[ a^3 - 3a^2b + 3ab^2 - b^3 = (a - b)^3 \]
   \[ a^2 - b^2 = (a + b)(a - b) \]
   \[ a^3 + b^3 = (a + b)(a^2 - ab + b^2) \]
   \[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \]

9. **Quadratic Formula**
   If \( ax^2 + bx + c = 0 \), then
   \[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

10. **Units Digits Patterns** \( (a^1, a^2, a^3..) \)
   
   \begin{align*}
   1 & \rightarrow 1 & 6 & \rightarrow 6 \\
   2 & \rightarrow 2, 4, 8, 6 & 7 & \rightarrow 7, 9, 3, 1 \\
   3 & \rightarrow 3, 9, 7, 1 & 8 & \rightarrow 8, 4, 2, 6 \\
   4 & \rightarrow 4, 6 & 9 & \rightarrow 9, 1 \\
   5 & \rightarrow 5
   \end{align*}

11. **Geometry**

   **General**
   Things that may be assumed from a geometric figure:
   1. Lines that appear to be straight are straight.
   2. Points of intersection are depicted accurately.
   3. Points shown on a line are collinear.
   4. All the points in the figure are coplanar unless specified otherwise.
   5. Relative positions of points are accurate.

   Some that may not be assumed from a given geometric figure:
   1. Exact measurement or relative size of figures.
   2. Congruence or similarity.
   3. Parallel or perpendicular lines.
Miscellaneous

- Number of diagonals in an \( n \)-gon \( \rightarrow \frac{n(n-3)}{2} \)
- Sum of measures of internal angles in an \( n \)-gon \( \rightarrow 180^\circ(n-2) \)
- Sum of measures of external angles in an \( n \)-gon \( \rightarrow 360^\circ \)
- If the ratio of the same linear dimension of two similar two-dimensional figures is \( k \), then the ratio of their areas is \( k^2 \)
- If the ratio of the same linear dimension of two similar three-dimensional figures is \( k \), then the ratio of their volumes is \( k^3 \)

Circle

- Area=\( \pi r^2 \), Circumference=\( 2\pi r \), where \( r \) is the radius

Triangle

- Area=\( \frac{1}{2}bh \), where \( b \) is a side (the base), and \( h \) is the length of the altitude drawn to that side (the height)
- Area=\( \sqrt{s(s-a)(s-b)(s-c)} \), where \( a, b, c \) are the side lengths and \( s \) is the semiperimeter (Heron’s Formula)
- Area=\( \frac{1}{2}ab \sin C \), where \( a, b \) are two sides and \( C \) is the angle formed by those sides
- Area=\( rs \), where \( r \) is the inradius of the triangle and \( s \) is the semiperimeter
- Area=\( \frac{3}{4}s\sqrt{3} \), where \( s \) is the length of a side of an equilateral triangle

Quadrilaterals

- Parallelogram
  - Area=\( bh \), where \( b \) is the base and \( h \) is the height of the parallelogram
- Rectangle
  - Area=\( lw \), Perimeter=\( 2(l+w) \), where \( l \) and \( w \) are the length and width of the rectangle
- Square
  - Area=\( s^2 \), where \( s \) is the side length
  - Area=\( d^2/2 \), where \( d \) is the diagonal
- Rhombus
  - Area=\( \frac{1}{2}d_1d_2 \), where \( d_1 \) and \( d_2 \) are the two diagonals of the rhombus

Prism

- Volume=\( Bh \), where \( B \) is the area of the base and \( h \) is the height
- Surface area=\( 2B + Ph \), where \( B \) is the area of the base, \( P \) is the perimeter of the base, and \( h \) is the height of the prism

Pyramid

- Volume=\( \frac{1}{3}Bh \), where \( B \) is the area of the base and \( h \) is the height
Rectangular Prism

- Volume = \(lwh\)
- Surface area = \(2(lw + wh + lh)\)
- Length of space diagonal = \(\sqrt{l^2 + w^2 + h^2}\), where \(l\), \(w\), and \(h\) are the lengths of the edges of the prism

Cube

- Volume = \(e^3\)
- Surface Area = \(6e^2\)
- Length of Space Diagonal: \(e\sqrt{3}\), where \(e\) is the edge length

Cylinder

- Volume = \(\pi r^2h\)
- Surface Area = \(2\pi rh + 2\pi r^2\), where \(r\) is the radius and \(h\) is the height

Cone

- Volume = \(\frac{1}{3}\pi r^2h\)
- Surface Area = \(\pi r^2 + \pi r\sqrt{r^2 + h^2}\)

Sphere

- Volume = \(\frac{4}{3}\pi r^3\)
- Surface Area = \(4\pi r^2\), where \(r\) is the radius

12. Triangles

Triangle Inequality
\(a + b < c\), where \(a\) and \(b\) are the shorter sides and \(c\) is the largest side

Pythagorean Theorem
\(a^2 + b^2 = c^2\), where \(a\) and \(b\) are the legs and \(c\) is the hypotenuse of a right triangle

Law of Cosines
\(c^2 = a^2 + b^2 - 2ab\cos C\), where \(a\), \(b\), and \(c\) are the sides of a triangle, and \(C\) is the measure of the angle formed by sides \(a\) and \(b\)

Congruency and Similarity Theorems

- Congruency
  For any triangles: SSS, SAS, ASA, AAS
  For right triangles: HL, LL, SA
- Similarity
  For any triangles: AA, SAS, SSS

Pythagorean Triples

- 3, 4, 5
- 5, 12, 13
- 8, 15, 17
- 7, 24, 25
- 12, 35, 37
- 9, 40, 41
- \(ka, kb, kc\), where \((a, b, c)\) is a Pythagorean triple and \(k\) is any positive real number
13. Trigonometry

Basic trig function mnemonic device: “SOHCAHTOA”

Definitions of trigonometric functions

\[
\begin{align*}
\sin \theta &= \frac{\text{opposite}}{\text{hypotenuse}} \\
\cos \theta &= \frac{\text{adjacent}}{\text{hypotenuse}} \\
\tan \theta &= \frac{\text{opposite}}{\text{adjacent}} = \frac{\sin \theta}{\cos \theta} \\
\sec \theta &= \frac{1}{\cos \theta} \\
\csc \theta &= \frac{1}{\sin \theta} \\
\cot \theta &= \frac{1}{\tan \theta}
\end{align*}
\]

Values of trigonometric functions for common angle measures

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>0°</th>
<th>30°</th>
<th>45°</th>
<th>60°</th>
<th>90°</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sin \theta )</td>
<td>0</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>1</td>
<td>( \infty )</td>
</tr>
<tr>
<td>( \cos \theta )</td>
<td>1</td>
<td>( \frac{\sqrt{3}}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
</tr>
<tr>
<td>( \tan \theta )</td>
<td>0</td>
<td>( \frac{\sqrt{3}}{3} )</td>
<td>1</td>
<td>( \sqrt{3} )</td>
<td>1</td>
</tr>
</tbody>
</table>

14. Some Approximations

- \( \pi \approx 3.14159 \)
- \( \sqrt{2} \approx 1.414 \)
- \( \sqrt{3} \approx 1.732 \)
- \( \sqrt{5} \approx 2.236 \)

15. Arithmetic Sequences

- \( a_1 + a_2 + \cdots + a_n = \frac{n(a_1 + a_n)}{2} \)
- \( 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} \)
- \( 1 + 3 + 5 + \cdots + (2n + 1) = n^2 \)
- \( 2 + 4 + 6 + 8 + \cdots + (2n) = n(n + 1) \)

16. Conversion Factors

<table>
<thead>
<tr>
<th>Length and Area</th>
<th>Volume</th>
<th>Metric Prefixes</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 in. = 1 ft.</td>
<td>3 ts. = 1 tbsp.</td>
<td>kilo- (10^3)</td>
</tr>
<tr>
<td>3 ft. = 1 yd.</td>
<td>2 tbsp. = 1 fl. oz.</td>
<td>hecto- (10^2)</td>
</tr>
<tr>
<td>5280 ft. = 1 mi.</td>
<td>8 fl. oz = 1 cup</td>
<td>deca- (10^1)</td>
</tr>
<tr>
<td>1760 yd. = 1 mi.</td>
<td>2 cups = 1 pt.</td>
<td>-</td>
</tr>
<tr>
<td>2.54 cm = 1 in.</td>
<td>2 pt. = 1 qt.</td>
<td>deci- (10^{-1})</td>
</tr>
<tr>
<td>640 acres = 1 mi.²</td>
<td>4 qt. = 1 gal.</td>
<td>centi- (10^{-2})</td>
</tr>
<tr>
<td>Weight</td>
<td></td>
<td>milli- (10^{-3})</td>
</tr>
<tr>
<td>16 oz. = 1 lb.</td>
<td>1 mL = 1 cm³</td>
<td></td>
</tr>
</tbody>
</table>